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A Theoretical Analysis on Transmission Characteristics of Semiconductor Lasers

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Abstract—**In this paper, on the basis of classical control theory transmission characteristics of semiconductor lasers will be analyzed. In the case of small-signals semiconductor lasers is considered as an isolated linear system and the rate equation describing its physical process is linearized; four kinds of transmission functions showing its transmission characteristics have been obtained by using network theory; response characteristics to optical-electrical input signals and corresponding equivalent network are then given according to the transmission functions, and transmission characteristics are, in turn, analyzed and synthesized according to the transmission functions and the equivalent network.**

I. Introduction

A great number of studies on transmission characteristics of semiconductor lasers have already been reported [1]-[3]. However, the analysis for its transmission characteristics often needs to solve complicated rate equations first; this sets forth before us a problem: whether a series of mathematical modes and corresponding equivalent networks can be established in a semiconductor laser system as that in other semiconductor devices. If it can be done, not only will the analysis process be simplified, but also the analysis and synthesis for the transmission characteristics can be brought into the well-established network theory.

In view of the control theory, semiconductor lasers can be considered as a relatively isolated system, merely controlled by external signals through "input" and giving its response to external signals through "output." If this system is a linear system, the excitation function is $M(t)$, and output $Q(t)$, then

$$\sum_{i=0}^{n} a_i Q^{(i)}(t) = \sum_{j=0}^{m} b_j M^{(j)}(t). \qquad (1)$$

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Through the Laplace transform, we obtain the equation

$$Q(p) \cdot \sum_{i=0}^{n} a_i p^i = M(p) \cdot \sum_{j=0}^{m} b_j p^j. \qquad (2)$$

Hence,

$$Q(p) = G(p) \cdot M(p) \qquad (3)$$

where

$$G(p) = \sum_{j=0}^{m} b_j p^j \Big/ \sum_{i=0}^{n} a_i p^i. \qquad (4)$$

$G(p)$ represents the transmission characteristics function of the system.

Supposing $p = j\omega$, we get the steady-state frequency response $G(j\omega)$; when we have the inverse Laplace transform of $G(p)$, the impulse response of system $G(t)$ is obtained, and the corresponding equivalent network of the system can be given according to the pole distribution of $G(p)$. Then, the output response characteristics of the system can be controlled by changing the excitation function and the parameters of the system itself, so the system can be synthesized according to the required response characteristics.

Based on the above consideration, in the case of small signal, the rate equations describing the physical process of semiconductor lasers are normalized and linearized in this paper And four kinds of transmission functions showing transmission characteristics of semiconductor lasers are obtained through the Laplace transform. By the use of network theory, and according to the pole characteristics of the transmission function, semiconductor lasers are classified into double-pole systems with equivalent or unequivalent real number poles, or with conjugate complex number poles, and the corresponding equivalent network model is given. According to the transmission function and the equivalent network, transmis-

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sion characteristics of semiconductor lasers are analyzed and synthesized.

II. Mathematical Model

Assuming that the active region of a semiconductor laser is made of uniform and p-type material with only one oscillation mode in the cavity, and the photon density distribution is uniform, then the rate equations for electron density n and photon density s are [1]-[3]

$$\frac{dn(t)}{dt} = \frac{j_e(t)}{d_0} - g(n)s(t) - \frac{n(t)}{\tau_e} \tag{5}$$

$$\frac{ds(t)}{dt} = \frac{j_p(t)}{d_0} + g(n)s(t) + \eta_i c\frac{n(t)}{\tau_e} - \frac{s(t)}{\tau_p} \tag{6}$$

where $j_e(t)$ is injected electron density; $j_p(t)$ is the photon density injected into the oscillation mode; d_0 is the active layer thickness; τ_e is the electron lifetime; τ_p is the photon lifetime; $g(n)$ is the stimulated emission gain; η_i is the internal quantum efficiency; and c is the percentage of spontaneous emission photon numbers filled into the oscillation mode [4].

There usually exists a nonlinearity between stimulated emission gain $g(n)$ and active region electron density n. For analytical convenience, linear approximation can be taken at the threshold electron density n_{th}. Set $g(n) = a(n - n')$, where n' is the intersection electron density during linear approximation, and a is the proportional constant. Taking $g/g_{\text{th}} = (n - n')/(n_{\text{th}} - n')$ and $g_{\text{th}} = 1/\tau_p$, and multiplying (5) and (6) on both sides by $\tau_e/(n_{\text{th}} - n')$, respectively, the relations

$$\frac{d\left(\frac{n - n'}{n_{\text{th}} - n'}\right)}{d\left(\frac{t}{\tau_e}\right)} = \left(\frac{j_e - j'}{j_{\text{th}} - j'}\right) - \left(\frac{n - n'}{n_{\text{th}} - n'}\right)\left(\frac{\tau_e}{\tau_p}\cdot\frac{s}{n_{\text{th}} - n'}\right) - \left(\frac{n - n'}{n_{\text{th}} - n'}\right) \tag{7}$$

$$\frac{d\left(\frac{\tau_e}{\tau_p}\cdot\frac{s}{n_{\text{th}} - n'}\right)}{d\left(\frac{t}{\tau_e}\right)} = \left(\frac{\tau_e}{\tau_p}\right)\left[\left(\frac{j_p - \eta_i c j'}{j_{\text{th}} - j'}\right) + \left(\frac{n - n'}{n_{\text{th}} - n'}\right)\left(\frac{\tau_e}{\tau_p}\cdot\frac{s}{n_{\text{th}} - n'}\right) + \eta_i c\left(\frac{n - n'}{n_{\text{th}} - n'}\right) - \left(\frac{\tau_e}{\tau_p}\cdot\frac{s}{n_{\text{th}} - n'}\right)\right] \tag{8}$$

are obtained.

Given

$$T = t/\tau_e,\ j_{\text{th}} = d_0 n_{\text{th}}/\tau_e,\ j' = d_0 n'/\tau_e,$$
$$K = \tau_e/\tau_p,\ D = \eta_i c,\ N(T) = (n - n')/(n_{\text{th}} - n'),$$
$$S(T) = KS/(n_{\text{th}} - n'),\ J(T) = (j_e - j')/(j_{\text{th}} - j'),$$
$$H(T) = (j_p - j')/(j_{\text{th}} - j'),$$

and substituting them into the above equations, we obtained the normalized rate equations

$$\frac{dN}{dT} = J - NS - N \tag{9}$$

$$\frac{dS}{dT} = K[H + (N - 1)S + DN]. \tag{10}$$

Under the steady-state condition, $J = J_0$, $H = H_0$, $dN/dT = 0$, $dS/dT = 0$, from (9) and (10), we have

$$N_0 = \{(J_0 + H_0) - [\sqrt{(J_0 + H_0 + 1)^2 - 4(1 - D)J_0} - 1]\}/2(1 - D) \tag{11}$$

$$S_0 = \{(J_0 + H_0) + [\sqrt{(J_0 + H_0 + 1)^2 - 4(1 - D)J_0} - 1]\}/2. \tag{12}$$

Under the condition of small signal superposition,

$$J(T) = J_0 + J_m(T), J_m(T) \ll J_0 \tag{13}$$

$$H(T) = H_0 + H_m(T), H_m(T) \ll H_0 \tag{14}$$

$$N(T) = N_0 + N_m(T), N_m(T) \ll N_0 \tag{15}$$

$$S(T) = S_0 + S_m(T), S_m(T) \ll S_0. \tag{16}$$

Substituting (13)-(16) into (9) and (10), by the use of the steady-state condition with the high order items omitted, the differential equations for small signal are given as

$$\frac{dN_m}{dT} = J_m - (1 + S_0)N_m - N_0 S_m \tag{17}$$

$$\frac{dS_m}{dT} = K[(N_0 - 1)S_m + (S_0 + D)N_m + H_m]. \tag{18}$$

Making the Laplace transform for (17) and (18), the relations

$$pN_m(p) = J_m(p) - (1 + S_0)N_m(p) - N_0 S_m(p) \tag{19}$$

$$pS_m(p) = K[(N_0 - 1)S_m(p) + (S_0 + D)N_m(p) + H_m(p)] \tag{20}$$

are obtained, in which $N_m(p)$, $S_m(p)$, $J_m(p)$, and $H_m(p)$ are the Laplace transform of $N_m(T)$, $S_m(T)$, $J_m(T)$, and $H_m(T)$, respectively.

From (19) and (20) we have

$$N_m(p) = G_{nj}(p)\cdot J_m(p) + G_{nh}(p)\cdot H_m(p) \tag{21}$$

$$S_m(p) = G_{sj}(p)\cdot J_m(p) + G_{sh}(p)\cdot H_m(p) \tag{22}$$

in which

$$G_{nj}(p) = [p + K(1 - N_0)]/\{p^2 + p[1 + S_0 + K(1 - N_0)] + K[1 + S_0 - N_0(1 - D)]\} \tag{23}$$

$$G_{nh}(p) = -KN_0/\{p^2 + p[1 + S_0 + K(1 - N_0)] + K[1 + S_0 - N_0(1 - D)]\} \tag{24}$$

$$G_{sj}(p) = K(S + D)/\{p^2 + p[1 + S_0 + K(1 - N_0)] + K[1 + S_0 - N_0(1 - D)]\} \tag{25}$$

$$G_{sh}(p) = K(p + 1 + S_0)/\{p^2 + p[1 + S_0 + K(1 - N_0)] + K[1 + S_0 - N_0(1 - D)]\}. \tag{26}$$

Expressing (21) and (22) with matrix, we get

$$\begin{pmatrix} N_m \\ S_m \end{pmatrix} = \begin{pmatrix} G_{nj} & G_{nh} \\ G_{sj} & G_{sh} \end{pmatrix}\begin{pmatrix} J_m \\ H_m \end{pmatrix}$$

where J_m and H_m are the electrical excitation function and optical excitation function of semiconductor lasers, respectively; N_m and S_m, respectively, are their electroresponse and optical response function; and G_{nj}, G_{nh}, G_{sj}, and G_{sh} are their transmission functions. We define G_{nj} as electrical transmission function, G_{nh} as photoelectrical conversion function, G_{sj} as electrophoto conversion function, and G_{sh} as optical transmission function. Supposing $p = 0$, from (23)-(26) the steady-state transmission coefficient is derived as

$$G_{nj}(0) = (1 - N_0)/[1 + S_0 - N_0(1 - D)] \quad (27)$$

$$G_{nh}(0) = -N_0/[1 + S_0 - N_0(1 - D)] \quad (28)$$

$$G_{sj}(0) = (S_0 + D)/[1 + S_0 - N_0(1 - D)] \quad (29)$$

$$G_{sh}(0) = (1 + S_0)/[1 + S_0 - N_0(1 - D)] \quad (30)$$

where $G_{nj}(0)$ is referred to as the electrical transmission coefficient; $G_{nh}(0)$ as the photoelectrical conversion coefficient, $G_{sj}(0)$ as the electrophoto conversion coefficient, and $G_{sh}(0)$ as the optical transmission coefficient.

From the above analysis we can see that transmission characteristics of semiconductor lasers only depend on the internal parameters (K, D) and external optoelectrical bias (H_0, J_0). $G(p)$ is actually the mathematical model indicating the transmission characteristics.

III. Transmission Characteristics

It can be found from network theory that transmission characteristics of semiconductor lasers depend on the pole distribution of the transmission function on the complex number plane. The response characteristics of frequency domain and time domain for semiconductor lasers will soon become clear after the pole is determined.

A. The Nature of Poles

For the convenience of the analysis we suppose

$$\xi = \tfrac{1}{2}\,[1 + S_0 + K(1 - N_0)]/\sqrt{K[1 + S_0 - N_0(1 - D)]} \quad (31)$$

$$\omega_0 = \sqrt{K[1 + S_0 - N_0(1 - D)]}\,. \quad (32)$$

Substituting them into (23)-(26), the following equations are obtained:

$$G_{nj}(p) = [p + K(1 - N_0)]/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (33)$$

$$G_{nh}(p) = -KN_0/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (34)$$

$$G_{sj}(p) = K(S_0 + D)/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (35)$$

$$G_{sh}(p) = K(p + 1 + S_0)/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (36)$$

with the characteristic equation

$$p^2 + 2\xi\omega_0 p + \omega_0^2 = 0. \quad (37)$$

Two poles of the transmission functions are obtained from (37) as

$$p_{1,2} = -\omega_0\xi \pm \omega_0\sqrt{\xi^2 - 1}. \quad (38)$$

We shall discuss three cases in the following: when $\xi < 1$,

$$p_{1,2} = -\omega_0\xi \pm j\omega_0\sqrt{1 - \xi^2} \quad (39)$$

the transmission function has two conjugate complex number poles, the semiconductor lasers correspond to the conjugate complex number double-pole system. Since $\mathrm{Re}[p_{1,2}] = -\omega_0\xi < 0$, and conjugate complex number poles distribute in the left half-part of the complex number plane, the time domain response of the system takes the form of an attenuation oscillation, while frequency domain response has resonance-like characteristics.

When $\xi = 1$

$$p_{1,2} = -\xi\omega_0 \quad (40)$$

the transmission function has two equivalent real number poles, semiconductor lasers, thus now correspond to an equivalent real number double-pole system. There is a damping form for the time domain response of the system, while there is no resonance-like characteristics for the frequency response of it because $p_{1,2} = -\xi\omega_0 < 0$, and the two poles distribute on the left half axis of the complex number plane.

When $\xi > 1$,

$$p_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1} = -\omega_0[\xi \mp \sqrt{\xi^2 - 1}] \quad (41)$$

and the transmission function has two unequivalent real number poles. Now the semiconductor lasers correspond to an unequivalent real number double-pole system. Since $p_{1,2} = -\omega_0[\xi \mp \sqrt{\xi^2 - 1}] < 0$, and both the two poles distribute on the left half axis of the complex number plane, there is a damping form for the time domain response of the system, while there is no resonance-like characteristics for the frequency response of it.

It can be found from this discussion that transmission characteristics are determined by ξ values which are dependent on their internal parameters (K, D) and external bias (J_0, H_0). However, when the internal parameter $K = 1 - D$,

$$\xi = \left[1 - \left(\frac{S_0 - N_0}{2 + S_0 - N_0}\right)^2\right]^{-1/2} \geqslant 1 \quad (42)$$

is obtained from (31).

In this case, semiconductor lasers are always a real number double-pole system. For practical semiconductor lasers $K = 10^3$-10^4 [7], $D = 10^{-6}$-10^{-3} [12]. So generally we always have $\xi \ll 1$, and these lasers always correspond to a conjugate complex number double-pole system. But if the values of the external bias (J_0, H_0) are changed, we can also have $\xi \geqslant 1$, and the original conjugate complex double-pole system is changed into a real number double-pole system (Fig. 1).

B. Steady-State Frequency Response

Set $p = j\omega$, and substitute it into (33)-(36); the steady-state frequency response is obtained as follows:

$$G_{nj}(j\omega) = G_{nj}(0)\cdot\left(1 + j\,\frac{\left(\dfrac{\omega}{\omega_0}\right)}{\dfrac{K(1 - N_0)}{\omega_0}}\right)\Bigg/\left[1 - \left(\frac{\omega}{\omega_0}\right)^2 + j2\xi\left(\frac{\omega}{\omega_0}\right)\right] \quad (43)$$

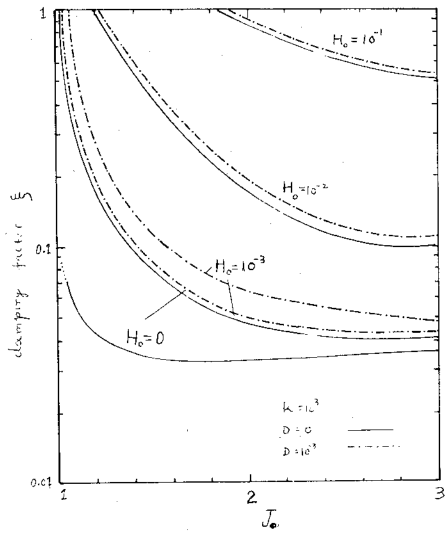

Fig. 1. The dependence of the damping factor on the external bias.

$$G_{sj}(j\omega) = G_{sj}(0) \cdot 1 \Big/ \left[1 - \left(\frac{\omega}{\omega_0}\right)^2 + j2\xi\left(\frac{\omega}{\omega_0}\right)\right] \tag{44}$$

$$G_{nh}(j\omega) = G_{nh}(0) \cdot 1 \Big/ \left[1 - \left(\frac{\omega}{\omega_0}\right)^2 + j2\xi\left(\frac{\omega}{\omega_0}\right)\right] \tag{45}$$

$$G_{sh}(j\omega) = G_{sh}(0) \cdot \left(1 + j\frac{\left(\frac{\omega}{\omega_0}\right)}{\frac{1+S_0}{\omega_0}}\right) \Big/ \left[1 - \left(\frac{\omega}{\omega_0}\right)^2 + j2\xi\left(\frac{\omega}{\omega_0}\right)\right]. \tag{46}$$

Its amplitude-frequency characteristics are as follows:

$$G_{nj}(\omega) = G_{nj}(0) \cdot \left[1 + \left(\frac{\left(\frac{\omega}{\omega_0}\right)}{\frac{K(1-N_0)}{\omega_0}}\right)^2\right]^{1/2} \Big/ \left\{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2\right\}^{1/2} \tag{47}$$

$$G_{sj}(\omega) = G_{sj}(0) \cdot 1 \Big/ \left\{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2\right\}^{1/2} \tag{48}$$

$$G_{nh}(\omega) = G_{nh}(0) \cdot 1 \Big/ \left\{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2\right\}^{1/2} \tag{49}$$

$$G_{sh}(\omega) = G_{sh}(0) \cdot \left[1 + \left(\frac{\left(\frac{\omega}{\omega_0}\right)}{\frac{1+S_0}{\omega_0}}\right)^2\right]^{1/2} \Big/ \left\{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2\right\}^{1/2}. \tag{50}$$

The phase-frequency characteristics are as follows:

$$\varphi_{nj}(\omega) = -\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_0}\right)}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right] + \tan^{-1}\left[\frac{\left(\frac{\omega}{\omega_0}\right)}{\frac{K(1-N_0)}{\omega_0}}\right] \tag{51}$$

$$\varphi_{sj}(\omega) = -\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_0}\right)}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right] \tag{52}$$

$$\varphi_{nh}(\omega) = -\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_0}\right)}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right] \tag{53}$$

$$\varphi_{sh}(\omega) = -\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_0}\right)}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right] + \tan^{-1}\left[\frac{\left(\frac{\omega}{\omega_0}\right)}{\frac{1+S_0}{\omega_0}}\right]. \tag{54}$$

Given $dG/d\omega = 0$, the limit value points of amplitude-versus-frequency curve are evaluated from (47)–(50). At $\xi \ll 1$, the limit value points are all ω_0; thus the maximum values for these curves are

$$G_{nj\max} = G_{nj}(0) \cdot \left[1 + \left(\frac{\omega_0}{K(1-N_0)}\right)^2\right]^{1/2} \Big/ 2\xi \tag{55}$$

$$G_{sj\max} = \frac{G_{sj}(0) \cdot 1}{2\xi} \tag{56}$$

$$G_{nh\max} = \frac{G_{nh}(0) \cdot 1}{2\xi} \tag{57}$$

$$G_{sh\max} = G_{sh}(0) \cdot \left[1 + \left(\frac{\omega_0}{1+S_0}\right)^2\right]^{1/2} \Big/ 2\xi. \tag{58}$$

It is obvious that when the angle frequency for the external input signal $\omega = \omega_0$, the steady-state frequency response characteristics exhibit resonance-like phenomena, and thus ω_0 is referred to as a resonance-like angular frequency of semiconductor lasers. Obviously, the values of ω_0 are also only dependent on internal parameters (K, D) and external bias (J_0, H_0). For a given semiconductor laser, when the external bias (J_0, H_0) is given, the characteristic parameters ω_0 and ξ are thus also specified.

C. Impulse Response

Making an inverse Laplace transform for (33)–(36), impulse response of semiconductor lasers can be obtained:

$$G_{nj}(T) = \left[1 + \left(\frac{A}{\omega_n}\right)^2\right]^{1/2} \cdot e^{-\sigma T} \cdot \cos(\omega_{nT} + \alpha) \tag{59}$$

$$G_{sj}(T) = K(S_0 + D) \cdot \frac{1}{\omega_n} \cdot e^{-\sigma T} \cdot \sin \omega_{nT} \tag{60}$$

$$G_{nh}(T) = -KN_0 \cdot \frac{1}{\omega_n} \cdot e^{-\sigma T} \cdot \sin \omega_{nT} \tag{61}$$

$$G_{sh}(T) = \left[1 + \left(\frac{A}{\omega_n}\right)^2\right]^{1/2} \cdot K \cdot e^{-\sigma T} \cdot \cos(\omega_n T - \alpha) \quad (62)$$

in which

$$A = \tfrac{1}{2}\,[1 + S_0 - K(1 - N_0)] \quad (63)$$

$$\omega_n = \omega_0 \sqrt{1 - \xi^2} = \omega_0 \sin\theta \quad (64)$$

$$\sigma = \omega_0 \xi = \omega_0 \cos\theta$$

$$\alpha = \tan^{-1}\left(\frac{A}{\omega_n}\right). \quad (65)$$

Here ω_n is referred to as the relaxation oscillation angular frequency, and σ as the relaxation oscillation attenuation coefficient. The limit values for every function are evaluated as follows from (59)-(62):

$$G_{nj\,\max}(T_0) = \left[1 + \left(\frac{A}{\omega_n}\right)^2\right]^{1/2} \cdot \cos\alpha, \qquad T_0 = 0 \quad (66)$$

$$G_{sj\,\max}(T_0) = K \cdot (S_0 + D) \cdot \frac{1}{\omega_n} \cdot e^{-\sigma T}, \qquad T_0 = \frac{1}{\omega_n}\theta \quad (67)$$

$$G_{nh\,\max}(T_0) = -KN_0 \frac{1}{\omega_n} e^{-\sigma T}, \qquad T_0 = \frac{1}{\omega_n}\theta \quad (68)$$

$$G_{sh\,\max}(T_0) = \left[1 + \left(\frac{A}{\omega_n}\right)^2\right]^{1/2} \cdot K \cdot \cos(-\alpha), \quad T_0 = 0. \quad (69)$$

It can be learned from the impulse response characteristics that there is a time difference $1/\omega_n\,\theta$ between the emergence of the limit values of electron density and that of photon density, which means there exists an exchanging process between electrical energy and optical energy. The limit values of electron density appear ahead of schedule for electrical exciting signal. The limit values of photon density appear ahead of schedule for optical exciting signal. After the impulse response $G(T)$ is determined, time domain response characteristics for any exciting signal can be obtained by using convolution integration [5].

D. Step Response

If the exciting signal is a unit step function, $H_m(p) = 1/p$ and $J_m(p) = 1/p$, step response functions are obtained from (33)-(36):

$$N_{mj}(p) = \frac{1}{p} \cdot [p + K(1 - N_0)]/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (70)$$

$$N_{mh}(p) = \frac{1}{p} \cdot (-KN_0)/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (71)$$

$$S_{mj}(p) = \frac{1}{p} \cdot K(S_0 + D)/(p^2 + 2\xi\omega_0 p + \omega_0^2) \quad (72)$$

$$S_{mh}(p) = \frac{1}{p} \cdot K(p + S_0 + 1)/(p^2 + 2\xi\omega_0 p + \omega_0^2). \quad (73)$$

Making an inverse Laplace transform of (70)-(73), step response of semiconductor lasers is obtained.

$$N_{mj}(T) = \frac{K(1 - N_0)}{\omega_0^2}\left[1 - \frac{\sqrt{1 + \alpha^2 - 2\alpha\sin\theta}}{\sin\theta} \cdot e^{-\sigma T} \cdot \sin(\omega_n T + \psi)\right] \quad (74)$$

$$N_{mh}(T) = -KN_0 \frac{1}{\omega_0^2}\left[1 - \frac{1}{\sin\theta} \cdot e^{-\sigma T} \cdot \sin(\omega_n T + \theta)\right] \quad (75)$$

$$S_{mj}(T) = K(S_0 + D) \cdot \frac{1}{\omega_0^2}\left[1 - \frac{1}{\sin\theta} \cdot e^{-\sigma T} \cdot \sin(\omega_n T + \theta)\right] \quad (76)$$

$$S_{mh}(T) = \frac{K(1 + S_0)}{\omega_0^2} \cdot \left[1 - \frac{\sqrt{1 + \beta^2 - 2\beta\sin\theta}}{\sin\theta} \cdot e^{-\sigma T} \cdot \sin(\omega_n T + \phi)\right] \quad (77)$$

where

$$\psi = \tan^{-1}\frac{\sin\theta}{\cos\theta - \alpha} = \tan^{-1}\frac{\sin\theta}{\cos\theta - \omega_0/K(1 - N_0)}$$

$$\phi = \tan^{-1}\frac{\sin\theta}{\cos\theta - \beta} = \tan^{-1}\frac{\sin\theta}{\cos\theta - \omega_0/(1 + S_0)}.$$

The first limit value points for the step responses are

$$T_{nj} = \frac{1}{\omega_n}(\theta - \psi) \quad (78)$$

$$T_{nh} = \frac{1}{\omega_n}\pi \quad (79)$$

$$T_{sj} = \frac{1}{\omega_n}\pi \quad (80)$$

$$T_{sh} = \frac{1}{\omega_n}(\theta - \phi). \quad (81)$$

Obviously, $T_{sj} > T_{nj}$, $T_{nh} > T_{sh}$, which indicates that for the step exciting signal response, the maximum values do not appear simultaneously. For electrical exciting signals, the peak values of electron density appear ahead of schedule; for optical exciting signals, the peak values of photon density appear ahead of schedule. Alter step response is determined by the response of semiconductor lasers to any exciting signal can be obtained by repeat integration [5].

E. Transmission Characteristics of Semiconductor Lasers with Electrical Bias

When $H_0 = 0, D = 0, J_0 > 1$, the relations

$$\xi = J_0/2\sqrt{K(J_0 - 1)} \quad (82)$$

$$\omega_0 = \sqrt{K(J_0 - 1)} \quad (83)$$

$$\sigma = J_0/2 \quad (84)$$

$$\omega_n = \sqrt{4K(J_0 - 1) - J_0^2}/2 \quad (85)$$

are derived from (31), (32), (64), and (65). Considering $\omega_0 = 2\pi f_0 \tau_e$ and $J_0 \simeq I_0/I_{th}$, the equations (82)-(85) can be written as

$$\xi = \frac{I_0}{I_{th}} \Big/ 2\sqrt{K\left(\frac{I_0}{I_{th}} - 1\right)} \quad (86)$$

$$f_0 = \frac{1}{2\pi\tau_e}\sqrt{K\left(\frac{I_0}{I_{th}} - 1\right)} \quad (87)$$

$$\sigma = \frac{1}{2} \frac{I_0}{I_{th}} \tag{88}$$

$$f_n = \frac{1}{4\pi\tau_e} \sqrt{4K\left(\frac{I_0}{I_{th}} - 1\right) - \left(\frac{I_0}{I_{th}}\right)^2}. \tag{89}$$

When $J_0 = 2$ is obtained supposing $d\xi/dJ_0 = 0$, there exists a minimum value for the damping factor ξ. In this case, (86)–(89) can be written as

$$\xi_{min} = \frac{1}{\sqrt{K}} \tag{90}$$

$$f_0 = \frac{\sqrt{K}}{2\pi\tau_e} \tag{91}$$

$$\sigma = 1 \tag{92}$$

$$f_n = \frac{\sqrt{K-1}}{2\pi\tau_e}. \tag{93}$$

The minimum damping factor means maximum resonance-like and relaxation oscillation of semiconductor lasers. Fig. 2 shows the response curve with the damping factor ξ as a parametric variable of semiconductor lasers under electrical bias.

IV. Equivalent Network

It is found from the discussion so far, that conventionally, semiconductor lasers correspond to a conjugate complex number double-pole system. Its mathematical model is rather similar to the well-known model for the *LCR* oscillation circuit. So we can use the network theory to have its network model equivalent to the network model of the *LCR* oscillation circuit, so that the active equivalent network of semiconductor laser can be obtained. Calculating (33)–(36) the relations

$$G_{nj}(p) = K_{nj} \cdot \left\{[p + K(1 - N_0)] \Big/ \left[p^2 \frac{1}{\omega_0^2} + p \frac{2\xi}{\omega_0} + 1\right]\right\} \tag{94}$$

$$G_{nh}(p) = K_{nh} \cdot \left\{1 \Big/ \left[p^2 \frac{1}{\omega_0^2} + p \frac{2\xi}{\omega_0} + 1\right]\right\} \tag{95}$$

$$G_{sj}(p) = K_{sj} \cdot \left\{1 \Big/ \left[p^2 \frac{1}{\omega_0^2} + p \frac{2\xi}{\omega_0} + 1\right]\right\} \tag{96}$$

$$G_{sh}(p) = K_{sh} \cdot \left\{(p + S_0 + 1) \Big/ \left[p^2 \frac{1}{\omega_0^2} + p \frac{2\xi}{\omega_0} + 1\right]\right\} \tag{97}$$

are obtained, where

$$K_{nj} = \frac{1}{\omega_0^2} = \frac{1}{K[1 + S_0 - N_0(1 - D)]} \tag{98}$$

$$K_{nh} = -\frac{KN_0}{\omega_0^2} = -\frac{N_0}{1 + S_0 - N_0(1 - D)} \tag{99}$$

$$K_{sj} = \frac{K(S_0 + D)}{\omega_0^2} = \frac{S_0 + D}{1 + S_0 - N_0(1 - D)} \tag{100}$$

$$K_{sh} = \frac{K}{\omega_0^2} = \frac{1}{1 + S_0 - N_0(1 - D)} \tag{101}$$

are referred to as the electrical transfer coefficient, opticoelectrical transfer coefficient, electrooptical transfer coefficient, and optical transfer coefficient, respectively.

There exists an *LCR* active network shown in Fig. 3 which is equivalent to the pole distribution according to (94)–(97) comparing the transmission function of the equivalent network with the relations (94)–(97), the active equivalent network parameters of semiconductor lasers can be obtained as

$$L = 1 \tag{102}$$

$$C = \frac{1}{K[1 + S_0 - N_0(1 - D)]} \tag{103}$$

$$R = K(1 - N_0) \tag{104}$$

$$r = 1 + S_0. \tag{105}$$

When $H_0 = 0$ and $D = 0$, (102)–(105) become the equivalent network parameters of electrical biased semiconductor lasers:

$$L = 1 \tag{106}$$

$$C = 1/K(J_0 - 1) \tag{107}$$

$$R = 0 \tag{108}$$

$$r = J_0 \tag{109}$$

with their transfer coefficients being

$$K_{nj} = 1/K(J_0 - 1) \tag{110}$$

$$K_{sj} = 1 \tag{111}$$

$$K_{nh} = -1/(J_0 - 1) \tag{112}$$

$$K_{sh} = 1/(J_0 - 1). \tag{113}$$

V. Analysis and Synthesis

According to the above mentioned equivalent network and transmission function analysis and synthesis on transmission characteristics of electrical biased semiconductor lasers can be given.

1) Under ordinary conditions, electrical biased semiconductor lasers correspond to the conjugate complex number double-pole system, whose steady-state frequency response has resonance-like characteristics, and whose step response has relaxation oscillation. In order to alleviate or eliminate these oscillation components, measures must be taken to raise the value of the damping factor ξ. Obviously, it can be seen from (31) and Fig. 2 that a change in the external optical bias H_0 and internal parameter (K, D) can result in a damping factor $\xi \geqslant 1$, and thus the semiconductor laser is made to change from the original conjugate complex number double-pole system into a real number double-pole system with the relaxation oscillation alleviated in the output response. This is actually the physical reality of the alleviating method mentioned in the literature [3], [6], [7].

2) Let the electrical excitation function be $J(p)$; after being transformed through the transfer network with transmission function $g(p)$, it is transferred into an electrical biased semiconductor laser, with the optical response function being

$$S(p) = J(p) \cdot g(p) \cdot G_{sj}(p). \tag{114}$$

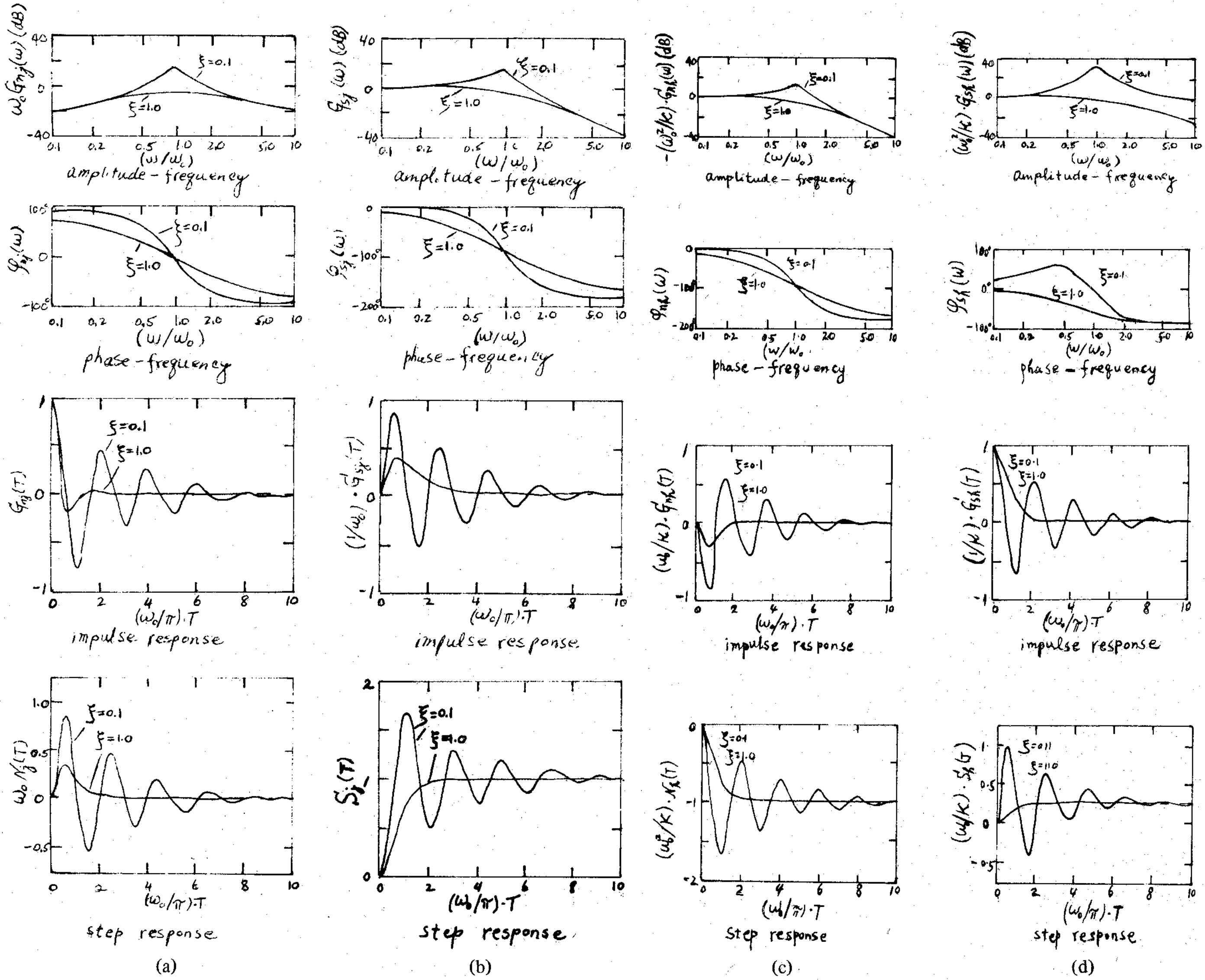

Fig. 2. Transmission characteristics of semiconductor lasers under electrical bias. (a) Electrical transmission characteristics. (b) Electrophoto conversion characteristics. (c) Photoelectrical conversion characteristics. (d) Optical transmission characteristics.

If it is required that the semiconductor laser response to a modulating signal without distortion, we require

$$g(p)G_{sj}(p) = 1,$$

that is, the transmission function is required to transform into

$$g(p) = [G_{sj}(p)]^{-1} = G_{sj}^{-1}(0) \cdot \left[p^2 \frac{1}{\omega_0} + p \frac{2\xi}{\omega_0} + 1\right]. \quad (115)$$

Obviously, this is a transfer network with "valley" frequency characteristics. According to the network theory, several transformation networks approximate to this transmission feature can be synthesized by dint of given parameters of the semiconductor laser. The network given by the literature [1] is one of the examples.

3) Suppose that the above-mentioned transfer network is a low-pass network whose transmission function runs as follows:

$$g(p) = \frac{1}{p + \omega_e} = \frac{1}{p + a\omega_0} \qquad (a \ll 1) \quad (116)$$

where ω_e is the 3 dB roll-off angular frequency of the low-pass network. The low-pass network limitations result in the angular frequency of the exciting signal $\omega \ll \omega_0$, hence the transmission function of the semiconductor laser $G_{sj}(p) \simeq G_{sj}(0)$. Now the response function is obtained from (114) as

$$S(p) = J(p) \cdot g(p) \cdot G_{sj}(0) = J'(p) \cdot G_{sj}(0). \quad (117)$$

With respect to the input signal function $J'(p)$, the semiconductor laser can verify a transmission without distortion. If $J(p) = 1/p$, then from (117) we can obtain

$$S(T) = G_{sj}(0) \cdot \frac{1}{a\sqrt{K(J_0 - 1)}} \cdot [1 - \exp(-a\sqrt{K(J_0 - 1)} \cdot T]. \quad (118)$$

Therefore, raising the electrical bias J_0 will make the semiconductor lasers better approach the transmission without distortion [8].

4) Let the electrical signal function be $J(p) = 1/p$, the opti-

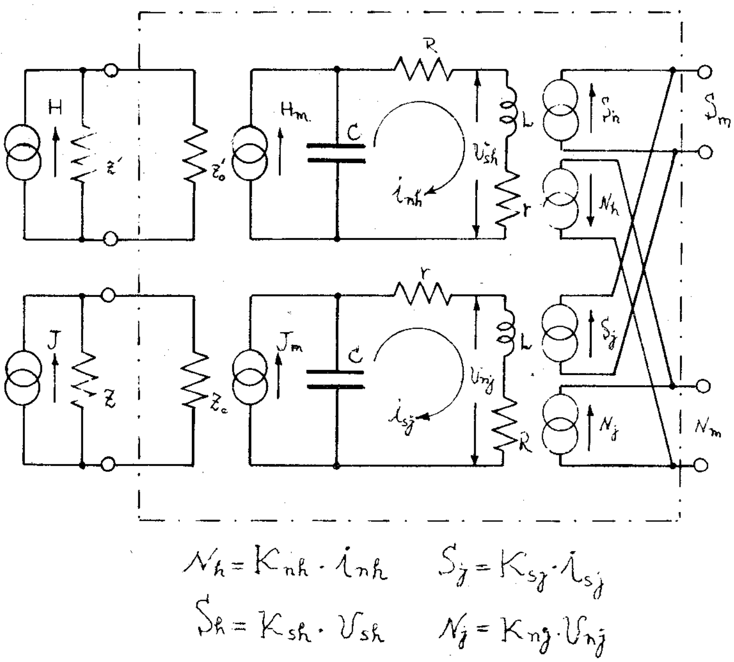

Fig. 3. The active equivalent network of semiconductor lasers.

cal signal function be $H(p) = \alpha \cdot 1/p \cdot e^{-pT_0}$, α be the attenuation coefficient of the optical signal intensity, and T_0 be the relaxation time of the optical signal with respect to electrical signal. The optical response function of the semiconductor laser can be obtained from the equivalent network as

$$S(p) = S_j(p) + S_h(p) = \frac{1}{p}\left[G_{sj}(p) + G_{sh}(p) \cdot e^{-pT_0}\right]. \quad (119)$$

Given $pT_0 \ll 1$, now $e^{-pT_0} \simeq 1 - pT_0$, substitute into the above equation, we obtain

$$S(p) = \frac{1}{p}\left\{\left(1 + \frac{J_0}{J_0 - 1}\alpha\right) \cdot \frac{\left[1 + \dfrac{\frac{\alpha}{J_0 - 1}(1 - J_0T_0)}{1 + \frac{J_0}{J_0 - 1}\alpha}p - \dfrac{\frac{\alpha}{J_0 - 1}J_0}{1 + \frac{J_0}{1 - \alpha}\alpha}p^2\right]}{\left[p^2\dfrac{1}{K(J_0 - 1)} + p\dfrac{J_0}{K(J_0 - 1)} + 1\right]}\right\}. \quad (120)$$

Let

$$\frac{\frac{\alpha}{J_0 - 1}(1 - J_0T_0)}{1 + \frac{J_0}{J_0 - 1}\alpha} = \frac{J_0}{K(J_0 - 1)}. \quad (121)$$

We obtain

$$T_0 = \frac{\alpha K(J_0 - 1) - J_0^2(1 + \alpha) + J_0}{K\alpha(J_0 - 1)J_0} \simeq \frac{(K\alpha - J_0)}{K\alpha}. \quad (122)$$

Substituting into (120), we obtain

$$S(p) = \frac{1}{p}\left\{\left(1 + \frac{J_0}{J_0 - 1}\alpha\right) \cdot \left[1 - \frac{\phi p^2}{p^2\dfrac{1}{K(J_0 - 1)} + p\dfrac{J_0}{K(J_0 - 1)} + 1}\right]\right\} \quad (123)$$

where

$$\phi = \frac{\alpha T_0}{(1 + \alpha)J_0 - 1} + \frac{1}{K(J_0 - 1)} \simeq \frac{\alpha}{J_0 - 1} - \frac{1}{K}. \quad (124)$$

A better choice of α can lead to $\phi \ll 1$; hence

$$S(T) \simeq \left(1 + \frac{J_0}{1 - J_0}\alpha\right) \cdot 1(T). \quad (125)$$

Now the semiconductor laser approximately verifies a transmission without distortion. If the optical signal is taken from the optical output of the semiconductor laser itself, this is actually the so-called optical feedback method [9]. However, because now the feedback optical signal is no longer the above-mentioned step form, but the relaxation oscillation form, the effect is not so significant as mentioned above.

5) From the transmission function and equivalent network, it can be seen that the semiconductor laser has amplification function and detection function with respect to optical signal [10], [11]. From (28), (30) and (11), (12) we can obtain the steady-state optical transmission coefficient and opticoelectrical conversion coefficient of semiconductor laser being, respectively,

$$G_{sh}(0) = \frac{1}{2}\left(\frac{J_0 + H_0 + 1}{\sqrt{(J_0 + H_0 + 1)^2 - 4(1 - D)J_0}} + 1\right) \quad (126)$$

$$G_{nh}(0) = \frac{-1}{2(1-D)} \left(\frac{J_0 + H_0 + 1}{\sqrt{(J_0 + H_0 + 1)^2 - 4(1-D)J_0}} - 1 \right). \tag{127}$$

With respect to electrical biased semiconductor laser, when $D = 0$, we have

$$G_{sh}(0) = J_0/(J_0 - 1) \tag{128}$$

$$G_{nh}(0) = -1/(J_0 - 1). \tag{129}$$

It shows that near the threshold value, the strongest steady-state amplification ability and detection ability appears. If resonance-like characteristics are used, optical amplification and detection of even higher efficiency can be obtained.

The theoretical analysis made on transmission characteristics of a uniform semiconductor laser is also suitable for a nonuniform semiconductor laser. The analysis and synthesis on its transmission characteristics will be given in another paper.

VI. Conclusion

This paper gives a theortical analysis on transmission characteristics of the semiconductor laser. Not only have the same results as those in the literature been obtained, but also these discrete results have been incorporated and unified. Our work shows that analysis and synthesis on transmission characteristics of semiconductor lasers can be carried out taking advantage of the well-established network theory.

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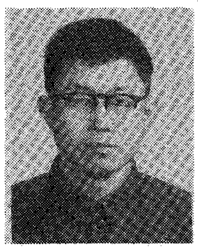

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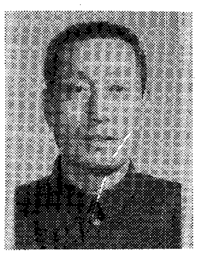

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